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CONSTRAINTS ON NON-NEWTONIAN GRAVITY FROM RECENT CASIMIR FORCE MEASUREMENTS

V.M. MOSTEPANENKO
Departamento de Física, Universidade Federal da Paraíba,
C.P. 5008, CEP 58059-970, João Pessoa, Pb-Brazil.
On leave from A. Friedmann Laboratory for Theoretical Physics, St.Petersburg, Russia

Abstract. Corrections to Newton's gravitational law inspired by extra dimensional physics and by the exchange of light and massless elementary particles between the atoms of two macrobodies are considered. These corrections can be described by the potentials of Yukawa-type and by the power-type potentials with different powers. The strongest up to date constraints on the corrections to Newton's gravitational law are reviewed following from the Eötvos- and Cavendish-type experiments and from the measurements of the Casimir and van der Waals force. We show that the recent measurements of the Casimir force gave the possibility to strengthen the previously known constraints on the constants of hypothetical interactions up to several thousand times in a wide interaction range. Further strengthening is expected in near future that makes Casimir force measurements a prospective test for the predictions of fundamental physical theories.

1. Introduction

It is common knowledge that the gravitational interaction is described on a different basis than all the other physical interactions. Up to the present there is no unified description of gravitation and gauge interactions of the Standard Model which would be satisfactory both physically and mathematically. Gravitational interaction persistently avoids unification with the other interactions. In addition, there is an evident lack of experimental data in gravitational physics. Newton's law of gravitation, which is also valid with high precision in the framework of the Einstein General Relativity Theory, is not verified with a sufficient precision at the separations less than 1 mm. Surprisingly, at the separations less than 1 μm corrections to the Newton's gravitational law are not excluded experimentally that are many orders of magnitude greater than the Newtonian force itself. What this means is the general belief, that the Newton's law of gravitation is obeyed up to Planckean separation distances, is nothing more than a large scale extrapolation. It is meaningful also that the Newton's gravitational constant G is determined with much less accuracy than the other fundamental physical constants. In spite of all attempts the results of recent experiments on the precision measurement of G are contradictory [1].

Prediction of non-Newtonian corrections to the law of gravitation comes from the extra dimensional unification schemes of High Energy Physics. According to this schemes, which go back to Kaluza [2] and Klein [3], the true dimensionality of physical space is larger than 3 with the extra dimensions being spontaneously compactified at the Planckean length-scale. At the separation distances several times larger than a compactification scale, the Yukawa-type corrections to the Newtonian gravitational potential do arise. This prediction would be of only academic interest if to take account of the extreme smallness of the Planckean length $l_{Pl} = \sqrt{G} \sim 10^{-33}$ cm (we use units with $\hbar = c = 1$) and the excessively high value of the Planckean energy $M_{Pl} = 1/\sqrt{G} = 10^{19}$ GeV. Recently, however, the low energy (high compactification length) unification schemes were proposed [4,5]. In the framework of these schemes the "true", multidimensional, Planckean energy takes a moderate value $M_* \sim 10^3$ GeV=1 TeV and the value of a compactification scale belongs to a submillimeter range. It is amply clear that in the same range the Yukawa-type corrections to the Newtonian gravitation are expected [6,7] and this prediction can be verified experimentally.

Much public attention given to non-Newtonian gravitation is generated not only by the extra dimensional physics. The new long-range forces which can be considered as corrections to the Newton's law of gravitation are produced also by the exchange of light and massless hypothetical elementary particles between the atoms of closely spaced macrobodies. Such particles (like axion, scalar axion, dilaton, graviphoton, moduli, arion etc.) are predicted by many extensions to the Standard Model and practically inavoidable in the modern theory of elementary particles and their interactions [8]. The long-range forces produced due to the exchange of hypothetical particles can be considered as some corrections to the Newton's gravitational law leading to the same phenomenological consequences as in the case of extra spatial dimensions.

The constraints on the constants characterizing the magnitude and interaction range of hypothetical forces are usually obtained from the gravitational experiments of Cavendish- and Eötvos-type. These experiments lead to the most strong constraints in the interaction range 10^{-5} m $< \lambda < 10^6$ km (see Ref. [9] and also some recent results in Refs. [10–13]). In nanometer and micrometer interaction range the best constraints on the constants of hypothetical interactions follow from the van der Waals and Casimir force measurements which provide the dominant background force at so small separations. The first results in this direction were obtained in Refs. [14,15] (see also Refs. [16,17] for details).

During the last years, the new experiments on measuring the Casimir force with an increased precision were performed [18–25]. They gave the possibility to considerably increase the strength of constraints on hypothetical interactions within a submillimeter interaction range [26–32]. Thus, from the measurement of the Casimir force by the use of an atomic force microscope [19–21] the strengthening of the previously known constraints up to 4500 times was obtained (the dynamical Casimir force measurements [23,33] lead to weaker constraints than those mentioned above). The increased experimental precision calls for a more accurate theory taking into account corrections to the Casimir force due to surface roughness, finite conductivity of the boundary metal and nonzero temperature. New constraints were obtained from a comparision between the more precise experimental data and improved theory (for the recent review of both experimental and theoretical developments in the Casimir effect see Ref. [34]).

In the present paper we report the most strong constraints on the hypothetical long-range interactions obtained from the Casimir effect. In Sec. 2 the hypothetical long-range forces are discussed originating from both extra dimensional physics and exchange of light elementary particles predicted by the unified gauge theories of fundamental interactions. In Sec. 3 the constraints from gravitational experiments are briefly summarized. Sec. 4 contains constraints following from Lamoreaux experiment [18] on measuring the Casimir force by means of a torsion pendulum. In Sec. 5 Mohideen et al experiments [19,20] on measurement of the Casimir force between an aluminum disk and a sphere by means of an atomic force microscope are considered. Sec. 6 is devoted to the results of Ederth experiment [22]. In Sec. 7 the most conclusive and reliable results are presented following from Mohideen et al experiment on measuring the Casimir force between gold surfaces [21]. In Sec. 8 reader will find the most recent results obtained from the lateral Casimir force measurement and from the new experiment using a microelectromechanical torsional oscillator. Sec. 9 contains conclusions and discussion.

Throughout the paper units are used in which $\hbar = c = 1$.

2. Origination of the hypothetical long-range interactions

The usual Newton's law of gravitation is only valid in a 4-dimensional space-time. If the extra dimensions exist, it will be modified by some corrections. In models with large but compact extra dimensions (like those proposed in Ref. [4]) the gravitational potential between two point particles with masses m_1 and m_2 separated by a distance $r \gg R_*$, where R_* is a compactification scale, is given by [6,7]

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \alpha_G e^{-r/\lambda}\right). \qquad (1)$$

The first term in the right-hand side of Eq. (1) is the Newtonian contribution, whereas the second term represents the Yukawa-type correction. Here G is the

Newton's gravitational constant, α_G is a dimensionless constant depending on the nature of extra dimensions and λ is the interaction range of a correction.

The dimensionless constant α_G in (1) depends on the nature of the extra dimensions. By way of example, for a toroidal compactification with all extra dimensions having equal size, $\alpha_G = 2n$ [6,7]. If extra dimensions have the topology of n-sphere $\alpha_G = n + 1$ [6,7].

In fact the search of corrections to Newtonian gravity, like in Eq. (1), is the simplest way to check the predictions of the models with low compactification scale. The thing is that, according to these models, all interactions and particles of the Standard Model are considered as living on a $(3 + 1)$-dimensional wall. They remain almost unchanged as this wall has a thickness only of order $M_*^{-1} \sim 10^{-17}$ cm in the extra dimensions. Only gravitational interaction penetrates freely into extra dimensions and can serve as a test for their existence.

At small separation distances $r \ll R_*$ the usual Newton's law of gravitation should be generalized to

$$V(r) = -\frac{G_{4+n} m_1 m_2}{r^{n+1}} \tag{2}$$

in order to preserve the continuity of the force lines in a $(4 + n)$-dimensional space-time. Here G_{4+n} is the underlying multidimensional gravitational constant connected with the usual one by the relation $G_{4+n} \sim G R_*^n$.

In fact the characteristic energy scale in multidimensional space-time is given by the multidimensional Planckean mass $M_* = 1/G_{4+n}^{1/(2+n)}$, and the compactification scale is given by [4]

$$R_* = \frac{1}{M_*} \left(\frac{M_{Pl}}{M_*} \right)^{2/n} \sim 10^{\frac{32}{n} - 17}\,\text{cm}, \tag{3}$$

where $M_{Pl} = 1/\sqrt{G}$ is the usual Planckean mass, $n \geq 1$, and $M_* \sim 10^3$ GeV as was told in Introduction. Then, for $n = 1$ (one extra dimension) one finds from Eq. (3) $R_* \sim 10^{15}$ cm. If to take into account that, as was shown in Refs. [6,7], $\alpha_G \sim 10$ and $\lambda \sim R_*$, this possibility must be rejected on the basis of the solar system tests of Newton's gravitational law [9]. If, however, $n = 2$ one obtains from Eq. (3) $R_* \sim 1\,$mm, and for $n = 3$ $R_* \sim 5\,$nm. For these scales the corrections of form (1) to Newton's gravitational law are not excluded experimentally.

The other type of multidimensional models considers noncompact but warped extra dimensions. In these models the leading contribution to the gravitational potential is given by [5,35]

$$U(r) = -\frac{G m_1 m_2}{r} \left(1 + \frac{2}{3k^2 r^2} \right), \tag{4}$$

where $r \gg 1/k$ and $1/k$ is the so-called warping scale. Here the correction to the Newton's gravitational law depends on the separation distance inverse proportionally to the third power of separation.

As was mentioned in Introduction, many extensions to the Standard Model predict the hypothetical long-range forces, distinct from gravitation and electromagnetism, caused by the exchange of light and massless elementary particles between the atoms of macrobodies. Under appropriate parametrization of the interaction constant these forces also can be considered as some corrections to the Newton's gravitational law. The velocity independent part of the effective potential due to the exchange of hypothetical particles between two atoms can be calculated by means of Feynman rules. For the case of massive particles with mass $\mu = 1/\lambda$ (λ is their Compton wavelength) the effective potential takes the Yukawa form

$$V_{Yu}(r) = -\alpha N_1 N_2 \frac{1}{r} e^{-r/\lambda}, \tag{5}$$

where $N_{1,2}$ are the numbers of nucleons in the atomic nuclei, α is a dimensionless interaction constant. If to introduce a new constant $\alpha_G = \alpha/(Gm_p^2) \approx 1.7 \times 10^{38}\alpha$ (m_p being a nucleon mass) and consider the sum of potential (5) and Newton's gravitational potential one returns back to the potential (1).

For the case of exchange of one massless particle the effective potential is just the usual Coulomb potential which is inverse proportional to separation. The effective potentials inverse proportional to higher powers of a separation distance appear if the exchange of even number of pseudoscalar particles is considered. The power-type potentials with higher powers of a separation are obtained also in the exchange of two neutrinos, two goldstinos or other massless fermions [16]. The resulting interaction potential acting between two atoms can be represented in the form [36]

$$U(r) = -\Lambda_l N_1 N_2 \frac{1}{r} \left(\frac{r_0}{r}\right)^{l-1}, \tag{6}$$

where $r_0 = 1\,$F=10^{-15} m is introduced for the proper dimensionality of potentials with different l, and Λ_l with $l = 1,\, 2,\, 3, \ldots$ are the dimensionless constants.

If to introduce a new set of constants $\Lambda_l^G = \Lambda_l/(Gm_p^2)$ and consider the sum of (6) and Newton's gravitational potential one obtains

$$U_l(r) = -\frac{Gm_1m_2}{r}\left[1 + \Lambda_l^G \left(\frac{r_0}{r}\right)^{l-1}\right]. \tag{7}$$

This equation represents the power-type hypothetical interaction as a correction to the Newton's gravitational law. The potential (4) following from the extra dimensional physics is obtained from Eq. (7) with $l = 3$. Note that the case $l = 3$ corresponds also to two arions exchange between electrons [16].

3. Constraints from gravitational experiments

Constraints on the corrections to Newton's gravitational law can be obtained from the experiments of Eötvos- and Cavendish-type. In the Eötvos-type experiments the difference between inertial and gravitational masses of a body is measured, i.e. the equivalence principle is verified. The existence of an additional hypothetical force which is not proportional to the masses of interacting bodies can lead to the appearance of the effective difference between inertial and gravitational masses. Therefore some constraints on hypothetical interactions emerge from the experiments of Eötvos type.

The typical result of the Eötvos-type experiments is that the relative difference between the accelerations imparted by the Earth, Sun or some laboratory attractor to various substances of the same mass is less than some small number. Many Eötvos-type experiments were performed (see, e.g., Refs. [37–40]). By way of example, in Ref. [39] the above relative difference of accelerations was to be less than 10^{-11}.

The results of the two precise Eötvos-type experiments can be found in Refs. [10,41]. They permit to obtain the best constraints on the constants of hypothetical long-range interactions inspired by extra dimensions or by the exchange of light and massless elementary particles (see Fig. 1).

The constraints under consideration can be obtained also from the Cavendish-type experiments. In these experiments the deviations of the gravitational force F from Newton's law are measured (see, e.g., Refs. [42–47]). The characteristic value of deviations in the case of two point-like bodies a distance r apart can be described by the parameter

$$\varepsilon = \frac{1}{rF}\frac{d}{dr}\left(r^2F\right), \tag{8}$$

which is equal exactly to zero in the case of pure Newton's gravitational force. For example, in Refs. [44,45] $|\varepsilon| \leq 10^{-4}$ at the separation distances $r \sim 10^{-2} - 1\,\text{m}$. This can be used to constrain the size of corrections to the Newton's gravitational law. The results of one of most recent Cavendish-type experiments can be found in Ref. [11].

Let us now outline the strongest constraints on the corrections to Newton's gravitational law obtained up to date from the gravitational experiments. The constraints on the parameters of Yukawa-type correction, given by Eq. (1), are presented in Fig. 1. In this figure, the regions of $(\lambda,\, \alpha_G)$-plane above the curves are prohibited by the results of the experiment under consideration, and the regions below the curves are permitted. By the curves 1 and 2 the results of the best Eötvos-type experiments are shown (Refs. [41] and [10], respectively). Curve 4 represents constraints obtained from the Cavendish-type experiment of Ref. [11]. At the intersection of curves 2 and 4 the better constraints are given by curve 3 following from the results of older Cavendish-type experiment of Ref. [47]. As is seen from Fig. 1, rather strong constraints on the Yukawa-type corrections to Newton's gravitational law ($\alpha_G < 10^{-5}$) are obtained only within the interaction range $\lambda > 0.1\,\text{m}$. With decreasing λ the strength of constraints falls off, so that

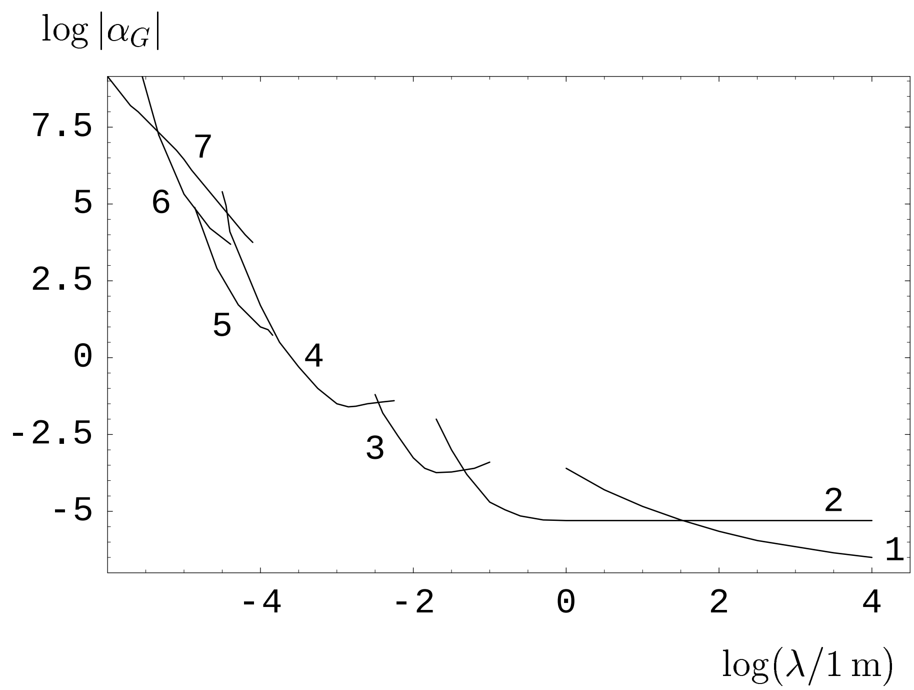

Figure 1. Constraints on the Yukawa-type corrections to Newton's gravitational law. Curves $1, 2$ follow from the Eötvos-type experiments, and curves 3–6 follow from the Cavendish-type experiments. The beginning of curve 7 shows constraints from the measurements of the Casimir force. Permitted regions on (λ, α_G)-plane lie beneath the curves.

at $\lambda = 0.1\,$mm $\alpha_G < 100$. By the beginning of curve 7 the constraints are shown following from the Casimir force measurements (see Sec. 4).

Recently two more precise Cavendish-type experiments were performed [12, 13] by the use of the micromachined torsional oscillator. They have permitted significantly increase the strength of constraints on α_G in the interaction range around (10–100) μm (see curve 5 [12] and curve 6 [13]).

Now we consider constraints on the power-type corrections to Newton's gravitational law given by Eq. (7). The best of them follow from the Eötvos- and Cavendish-type experiments. They are collected in Table 1.

For $l = 1,\ 2$ the constraints presented in Table 1 are obtained from the Eötvos-type experiments, and for $l = 3,\ 4,\ 5$ from the Cavendish-type ones. It is seen that the strength of constraints falls greatly with the increase of l.

TABLE I. Constraints on the constants of power-type potentials.

l	$\|\Lambda_l\|_{\max}$	$\|\Lambda_l^G\|_{\max}$	Source
1	6×10^{-48}	1×10^{-9}	Ref. [48]
2	2.4×10^{-30}	4×10^{8}	Ref. [10]
3	7×10^{-17}	1.2×10^{22}	Refs. [47, 49, 50]
4	7.5×10^{-4}	1.3×10^{35}	Refs. [45, 50]
5	1.2×19^{9}	2×10^{47}	Refs. [45, 50]

4. Constraints following from Lamoreaux experiment by means of torsion pendulum

As is seen from Sec. 3, for larger interaction distances the best constraints on the corrections to Newton's gravitational law follow from the Eötvos-type experiments and for lesser interaction distances from the Cavendish-type ones. With the further decrease of the characteristic interaction distance the strength of constraints following from the gravitational experiments greatly reduces. Within a micrometer separations, the Casimir and van der Waals force [17,51,52] becomes the dominant force between two macrobodies. As was shown in Ref. [14] for the case of Yukawa-type interactions with a micrometer interaction range and in Ref. [15] for the power-type ones, the measurements of the van der Waals and Casimir forces lead to the strongest constraints on non-Newtonian gravity (see the discussion about the Casimir effect as a test for non-Newtonian gravitation in Ref. [53]).

Currently a lot of precision experiments on the measurement of the Casimir and van der Waals force has been performed (see Ref. [34] for a review). As was mentioned in Introduction, the extensive theoretical study of different corrections to the Casimir force due to surface roughness, finite conductivity of a boundary metal and nonzero temperature gave the possibility to compute the theoretical value of this force with high precision. At the moment the agreement between theory and experiment at a level of 1% is achieved for the smallest experimental separation distances [34]. This permitted to obtain stronger constraints on the corrections to Newton's gravitational law from the results of the Casimir force measurements [26–32,54,55]. Here we briefly present the strongest constraints of this type starting from the first modern experiment performed by Lamoreaux [18].

In Ref. [18] the Casimir force between a spherical lens and a disk made of quartz (with the densities $\rho' = 2.23\times 10^3\,$kg/m^3 and $\rho = 2.4\times 10^3\,$kg/m^3, respectively) and coated by Cu and Au layers of thickness $\Delta_1 = \Delta_2 = 0.5\,\mu$m (with the densities $\rho_1 = 8.96\times 10^3$ kg/m^3, $\rho_2 = 19.32\times 10^3$ kg/m^3) was measured by the use of torsion pendulum. The disk radius was $L = 1.27\,$cm and a lens height and curvature radius were $H = 0.18\,$cm and $R = 12.5\,$cm, respectively.

The absolute error of force measurements in Ref. [18] was about $\Delta F = 10^{-11}\,$N for the separation range between a disk and a lens $1\,\mu$m $\leq a \leq 6\,\mu$m. In the limits of this error the theoretical expression for the Casimir force was confirmed

$$F^{(0)}(a) = -\frac{\pi^3}{360}\,\frac{R}{a^3}. \tag{9}$$

No corrections to Eq. (9) due to surface roughness, finite conductivity of the boundary metal or nonzero temperature were reported. These corrections, however, may not lie within the limits of the absolute error ΔF. By way of example, at $a = 1\,\mu$m roughness correction $\Delta_R F(a)$ may be around 12% of $F^{(0)}$ or even larger [26]. The finite conductivity correction $\Delta_{\delta_0} F(a)$ for gold surfaces at $a = 1\,\mu$m separation is 10% of $F^{(0)}$ [56]. (Note that ΔF is around 3% of $F^{(0)}$ at $a = 1\,\mu$m.) As to temperature correction $\Delta_T F(a)$, it achieves 174% of $F^{(0)}$ at the separation $a = 6\,\mu$m, where, however, ΔF is around 700% of $F^{(0)}$. For this reason, the constraints on the Yukawa-type interaction following from Lamoreaux experiment were found from the inequality [26]

$$|F_{th}(a) - F^{(0)}(a)| \leq \Delta F, \tag{10}$$

where F_{th} is the theoretical force value including $F^{(0)}$, all the corrections to it mentioned above, and also the hypothetical Yukawa-type interaction

$$F_{th}(a) = F^{(0)}(a) + \Delta_R F(a) + \Delta_{\delta_0} F(a) + \Delta_T F(a) + F_{Yu}(a) \tag{11}$$

(we remind that the sign of a finite conductivity correction is opposite to the sign of other corrections).

The hypothetical interaction in a configuration of a spherical lens above a disk was computed in Ref. [26]. For λ smaller or of order of separation a the result is given by

$$\begin{aligned} F_{Yu}(a) = & -4\pi^2 \alpha_G G \lambda^3 e^{-a/\lambda} R \\ & \times \left[\rho_2 - (\rho_2 - \rho_1)\, e^{-\Delta_2/\lambda} - \left(\rho_1 - \rho'\right) e^{-(\Delta_2+\Delta_1)/\lambda}\right] \\ & \times \left[\rho_2 - (\rho_2 - \rho_1)\, e^{-\Delta_2/\lambda} - (\rho_1 - \rho)\, e^{-(\Delta_2+\Delta_1)/\lambda}\right]. \end{aligned} \tag{12}$$

For larger λ, $F_{Yu}(a)$ was computed numerically [26].

The obtained constraints [26] are shown in Fig. 2 (curve 7,a for $\alpha_G > 0$ and curve 7,b for $\alpha_G < 0$). In this figure, the regions of (α_G, λ)-plane above the curves are prohibited, and the regions below the curves are permitted by the results of an experiment under consideration. By the curves 6 the results of the best Cavendish-type experiments are shown (Ref. [13]). Curve 8 represents constraints obtained from the Casimir force measurements between dielectrics

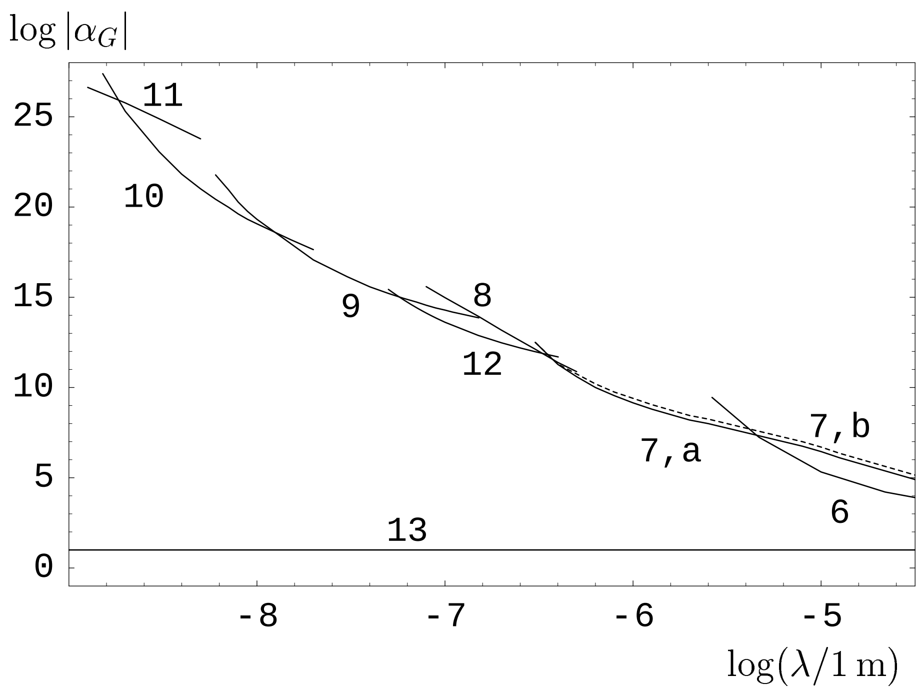

Figure 2. Constraints on the Yukawa-type corrections to Newton's gravitational law. Curves 8–10, 12 follow from the Casimir, and curve 11 from the van der Waals force measurements. The typical prediction of extra dimensional physics is shown by curve 13.

[16,17]. Line 13 demonstrates the typical prediction of extra dimensional theories. The strengthening of constraints given by curves 7,a and 7,b comparing curve 8 is up to a factor of 30 in the interaction range $2.2 \times 10^{-7}\,\mathrm{m} \leq \lambda \leq 5 \times 10^{-6}\,\mathrm{m}$ (a weaker result was obtained later in Ref. [29] where the corrections to the ideal Casimir force of Eq. (9) were not taken into account). This shows that the Casimir force measurements are competitive with the Cavendish-type experiments in a micrometer interaction range.

5. Constraints following from Mohideen et el experiments with Al surfaces by means of atomic force microscope

A major progress in obtaining more strong constraints on the Yukawa-type interactions within a nanometer range was achieved due to the measurements of the Casimir force by means of an atomic force microscope [19–21]. In Refs. [19,56] the results of the Casimir force measurement between a flat supphire disk ($L =$

$0.625\,\text{cm}$, $\rho = 4.0 \times 10^3\,\text{kg/m}^3$) and a polystyrene sphere ($R = 98\,\mu\text{m}$, $\rho' = 1.06 \times 10^3\,\text{kg/m}^3$) were presented in comparison with a complete theory taking into account the finite conductivity and roughness corrections. Temperature corrections are not essential in the separation range $0.12\,\mu\text{m} \leq a \leq 0.9\,\mu\text{m}$ used in Refs. [19,56]. The test bodies were coated by the aluminum layer ($\rho_1 = 2.7 \times 10^3\,\text{kg/m}^3$) of $\Delta_1 = 300\,\text{nm}$ thickness and Au/Pd layer ($\rho_2 = 16.2 \times 10^3\,\text{kg/m}^3$) of the thickness $\Delta_2 = 20\,\text{nm}$ (this latter was used to prevent the oxidation processes; it is almost transparent for electromagnetic oscillations of characteristic frequency). The absolute error of force measurements in Refs. [19,56] was $\Delta F = 2 \times 10^{-12}\,\text{N}$. In the limits of this error the theoretical expression for the Casimir force with corrections to it due to the surface roughness and finite conductivity was confirmed.

In the improved version of this experiment [20] the Au/Pd layer was made thiner ($\Delta_2 = 7.9\,\text{nm}$) and the other experimental parameters were as follows: $\Delta_1 = 250\,\text{nm}$, $L = 0.5\,\text{cm}$, $R = 100.85\,\mu\text{m}$, $100\,\text{nm} \leq a \leq 500\,\text{nm}$. Due to experimental improvements like the use of vibration isolation, lower systematic errors, independent measurement of a surface separation and smoother metal surface, the smaller absolute error of force measurement $\Delta F = 1.3 \times 10^{-12}\,\text{N}$ was achieved. In the limits of this error experimental data were in agreement with a complete theory.

To obtain constraints on hypothetical interactions, the hypothetical force was computed [27,28] with account of surface roughness contribution which is especially important at the closest separations

$$F_{Yu}(a) = \sum_i w_i F_{Yu}(a_i). \tag{13}$$

Here w_i are the probabilities for different values of a separation distance between the distorted surfaces, and F_{Yu} is given by Eq. (12). The values of w_i were found [56] on the basis of atomic force microscope measurements of surface roughness. The typical roughness heights were 40 nm and 20 nm (Refs. [19,56]) and 14 nm and 7 nm (Ref. [20]).

The constraints were obtained from the inequality

$$|F_{Yu}(a)| \leq \Delta F, \tag{14}$$

because the theoretical expression for the Casimir force with all corrections to it was confirmed experimentally unlike the case considered in Sec. 4.

Using the above experimental parameters of Refs. [19,56], the strengthening of constraints up to 140 times was obtained [27] as compared with the measurements of the van der Waals and Casimir force between dielectrics. The strengthening holds within the interaction range $5.9\,\text{nm} \leq \lambda \leq 100\,\text{nm}$.

Even stronger constraints were obtained [28] from the experiment of Ref. [20] in a wider interaction range $5.9\,\text{nm} \leq \lambda \leq 115\,\text{nm}$. The new constraints are up to 560 times stronger than the old ones given by curves 8 and 11 in Fig. 2 which follow from old mesurements of the Casimir and van der Waals force between dielectrics (the final constraint curve from the atomic force microscopy measurements will be obtained in Sec. 7).

The above constraints obtained on the basis of Refs. [19, 20, 56] are found for the closest separation distance a (120 nm in Refs. [19, 56] and 100 nm in Ref. [20]). If to decrease the minimal value of a, stronger constraints can be obtained. This is true also if the heavier metal coating is used (see below).

6. Constraints on hypothetical interactions following from Ederth experiment with two crossed cylinders

In Ref. [22] the Casimir force acting between two crossed quartz cylinders of 1 cm radius was measured (quartz density $\rho=\rho'=2.23\times 10^3$ kg/m^3). Each cylinder was coated by a layer of Au (density $\rho_1 = 18.88\times 10^3$ kg/m^3 and thickness $\Delta_1=200$ nm) and outer layer of hydrocarbon (density $\rho_2=0.85\times 10^3$ kg/m^3, thickness $\Delta_2=2.1$ nm). The absolute error of force measurements was $\Delta F = 10$ nN which is much larger than in the experiments discussed above. Within the limits of this error the theoretical expression for the Casimir force between cylinders was confirmed. The separation range between the cylinders was in the limits $20\,\text{nm}\le a\le 100\,\text{nm}$, i.e. the more close separations were achieved. The other experimental improvement of Ref. [22] lies in the use of smoother surfaces. The root mean square roughness of the cylindrical surfaces was decreased up to 0.4 nm.

There were also some disadvantages in the experiment of Ref. [22] as compared with the previous experiments. One of them is connected with the presence of hydrocarbon coating which complicates the independent measurement of the residual electrostatic force. The other disadvantage is a substantial deformation of the Au coating caused by the attractive forces in contact and by relatively soft glue used to support the Au layer. As a result, there is no independent and exact determination of surface separation in Ref. [22].

The constraints on the parameters of Yukawa-type interaction, following from the experiment of Ref. [22], were obtained from Eq. (14) in Ref. [30]. It was shown [30] that the hypothetical force between two cylinders, crossed at a right angle, is given once more by Eq. (12). Surface roughness contribution is not essential here as the roughness amplitude was considerably decreased.

The obtained constraints [30] are shown by curve 10 in Fig. 2 (by curve 11 the constraints following from the van der Waals force measurements between dielectrics are demonstrated). They are up to 300 times stronger than the previously known ones within the separation range $1.5\,\text{nm}\le\lambda\le 11\,\text{nm}$. This result was obtained at the closest separation distance $a=20$ nm.

7. Constraints on hypothetical interactions following from Mohideen et el experiments with gold surfaces

The most conclusive measurement of the Casimir force by means of atomic force microscope was performed between a sapphire disk and polystyrene sphere ($R = 95.65\,\mu$m) coated by Au layer of $\Delta_1 = 86.6\,$nm thickness [21]. No additional coating was used which added complexity to interpretation of experimental data of Refs. [19,20,22]. Some other improvements were implemented also in this experiment. Specifically, the root mean square amplitude of surface roughness was decreased up to $1.0 \pm 0.1\,$nm which is comparable with Ref. [22] (see the preceding section) but did not require additional hydrocarbon coating. The electrostatic forces were reduced to a value much smaller of the Casimir force at the shortest separation and used for an independent measurement of surface separation. Also the measurement was performed over smaller separations $62\,\text{nm} \leq a \leq 350\,\text{nm}$ than in previous measurements by means of atomic force microscope.

The absolute error of force measurements in Ref. [21], $\Delta F = 3.5 \times 10^{-12}\,$N, was a bit larger than that in Refs. [19,20]. This was caused by the poor thermal conductivity of the cantilever resulting from the thiner metal coating used. The increase of ΔF is, however, compensated for by the greater increase of the Casimir force at smaller separations.

Constraints on the constants of Yukawa-type interaction were obtained [31,57] from Eq. (14) using the agreement of experimental data with a theoretical Casimir force. Hypothetical force was computed by Eq. (12) having regard to $\Delta_2 = \rho_2 = 0$. The computational results are shown by curve 9 in Fig. 2. The obtained constraints are stronger up to 19 times, comparing the previous experiments using the atomic force microscope, within the interaction range $4.3\,\text{nm} \leq \lambda \leq 150\,\text{nm}$. If to compare with the experiment of Ref. [22], the constraints following from Mohideen et al experiment with Au surfaces prove to be the best ones in the interaction range $11\,\text{nm} \leq \lambda \leq 150\,\text{nm}$. As a consequence, the constraints, which are up to 4500 times more stringent than those from older Casimir and van der Waals force measurements between dielectrics, are obtained from the experiments by means of the atomic force microscope.

8. Constraints from measurements of the lateral Casimir force and from experiment using a microelectromechanical torsional oscillator

In 2002, the new physical phenomenon, the lateral Casimir force, was demonstrated first [24,25] acting between a sinusoidally corrugated gold plate and large sphere. This force acts in a direction tangential to the corrugated surface. The experimental setup was based on the atomic force microscope specially adapted for the measurement of the lateral Casimir force. The measured force oscillates sinusoidally as a function of the phase difference between the two corrugations in

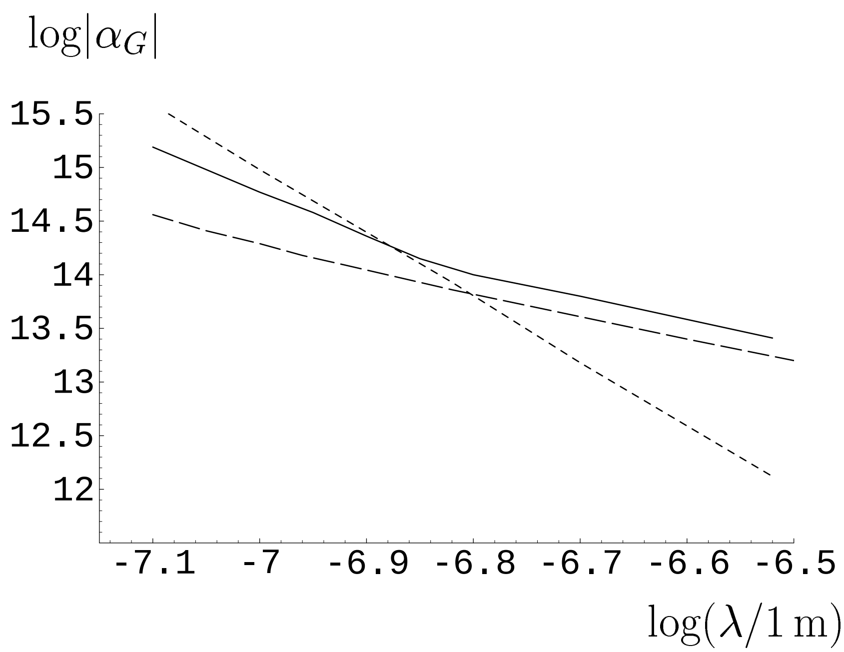

Figure 3. Constraints on the Yukawa-type corrections to Newton's gravitational law from the measurement of the lateral Casimir force between corrugated surfaces (solid curve). For comparison the short-dashed and long-dashed curves reproduce curves 8 and 9 of Fig. 2, respectively, obtained from the meausrements of the normal Casimir force between dielectrics and between gold surfaces.

agreement with theory with an amplitude of 3.2×10^{-13} N at a separation distance 221 nm. So small value of force amplitude measured with a resulting absolute error 0.77×10^{-13} N [25] with a 95% confident probability gives the opportunity to obtain constraints on the respective lateral hypothetical force which may act between corrugated surfaces.

The obtained constraints [25,32] are shown in Fig. 3 as the solid curve. In the same figure, the short-dashed curve indicates constraints obtained from the old Casimir force measurements between dielectrics (curve 8 in Fig. 2), and the long-dashed curve follows from the most precision measurement of the normal Casimir force between gold surfaces [21] (these constraints were already shown by curve 9 in Fig. 2). The constraints obtained by means of the lateral Casimir force measurement are of almost the same strength as the ones known previously in the interaction range 80 nm$<\lambda<$150 nm. However, with the increase of accuracy of the lateral Casimir force measurements more promising constraints are expected.

Recently one more experiment was performed on measuring the normal Casimir force. For this purpose the microelectromechanical torsional oscillator has been used. This permitted to perform measurements of the Casimir force between a sphere and a plate with an absolute error of 0.3 pN and of the Casimir pressure between two parallel plates with an absolute error of about 0.6 mPa [58,59] for separations $0.2 - 1.2\,\mu$m. As a result, the new constraints on the Yukawa-type corrections to Newton's law of gravitation were obtained [59] which are more than one order of magnitude stronger than the previously known ones within a wide interaction range from 56 nm to 330 nm. These constraints are shown by curve 12 in Fig. 2. It is notable that the constraints given by curve 12 almost completely cover the gap between the modern constraints obtained by means of an atomic force microscope and a torsion pendulum. Within this gap the constraints found from the old measurements of the Casimir force between dielectrics (curve 8 in Fig. 2) were the best ones. Now they are changed to the more precise and reliable constraints obtained from the Casimir force measurements between metals by means of a microelectromechanical torsional oscillator.

As is seen from Figs. 2, 3, the present strength of constraints is not sufficient to confirm or to reject the predictions of extra dimensional physics with the compactification scale $R_* < 0.1$ mm (line 13 in Fig. 2). However, Fig. 2 gives the possibility to set constraints on the parameters of light hypothetical particles, moduli, for instance. Such particles are predicted in superstring theories and are characterized by the interaction range from one micrometer to one centimeter [60].

9. Conclusions and discussion

In the above, the modern constraints on the constants of hypothetical interactions are reviewed following from the Casimir force measurements and gravitational experiments. As is evident from Fig. 2, for separations smaller than 10^{-4} m much work is needed to achieve the strength of Yukawa interaction predicted by extra dimensional physics ($\alpha_G \sim 10$). However, one should remember that Casimir force experiments are also sensitive to other non-extra dimensional effects such as exchange by light elementary particles which can lead to the Yukawa-type forces with $\alpha_G \gg 10$ (see Refs. [9,16,60]). Therefore, all experiments which can strengthen the constraints on constants of the Yukawa-type interaction are of immediate interest to both elementary particle physics and gravitation [61].

The following conclusions might be formulated. The idea of hypothetical interactions has gained recognition. New long-range forces additional to the usual gravitation and electromagnetism are predicted by the extra dimensional physics. They may be caused also by the exchange of light elementary particles predicted by the unified gauge theories of fundamental interactions.

The modern measurements of the Casimir force already gave the possibility to strengthen constraints on hypothetical long-range forces up to several thousand times in a wide interaction range from 1 nanometer to 100 micrometers.

Further strengthening of constraints on non-Newtonian gravity and other hypothetical long-range interactions from the Casimir effect is expected in the future. In this way the Casimir force measurements are quite competitive with the modern accelerator and gravitational experiments as a test for predictions of fundamental physical theories.

In near future we may expect to obtain the resolution of the problem are there exist large extra dimensions and the Yukawa-type corrections to Newtonian gravity at small distances.

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